

1 *Title:* **SNP Genotype Effects Model as an**
2 **Alternative to Animal Models**

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6 *Running Title:* **Genotype Effects Model**

7 **Summary**

8 The animal model in dairy cattle has become obsolete due to the pre-
9 selection of bull calves based on genomic EBVs that is occurring in many
10 countries. The bias caused by intense pre-selection permeates EBVs of all
11 animals via the additive relationship matrix and/or the genomics relationship
12 matrix, and is getting more severe the longer that pre-selection is employed.
13 There is little that can be done (in a statistical sense) to remove the pre-
14 selection bias. Goddard (2011) and Schaeffer (2011) both proposed changing
15 from animal models to models that estimate unbiased SNP genotype effects in
16 order to avoid the pre-selection biases. The details of such a model and the
17 procedures around it are presented using a small example.

18 *Keywords:* Animal model
19 Genotype effects model
20 Markers

21

22 Introduction

23 The individual cow model was described in Henderson's course notes as
24 early as 1967. Quaas and Pollak (1980) changed the name to animal model,
25 which has persisted. Animal models were first adopted in dairy cattle around
26 1987 (Wiggans and Misztal, 1987) for milk production in Ayrshire dairy cattle,
27 and 1988 for dairy conformation (Jamrozik and Schaeffer, 1988). Within five
28 years many countries were calculating animal model genetic evaluations for
29 dairy cattle traits. Kennedy et al. (1988) showed that the animal model,
30 using the additive genetic relationship matrix, could account for non-random
31 matings of bulls to cows, but the model still required that progeny groups of
32 each mating were a random sample of all genetically possible offspring. This
33 assumption is now violated due to preselection of male progeny on the basis of
34 particular marker genotypes and associated genomic estimated breeding values
35 (GEBV). Today, the progeny of each mating being grown and measured are
36 no longer a random sample of all potential progeny, but rather an intensely
37 selected set of progeny. Consequently, inclusion of selected offspring in an
38 animal model biases the evaluations of the sire and dam. Bias also affects
39 the EBVs of contemporaries of the selected progeny. From there the bias is
40 spread to the EBVs of all animals in the pedigree. The fact that the bias is
41 spread to every animal, means its effects are slightly muted over all. However,
42 rankings of animals can be affected, genetic trends can be overestimated, and
43 therefore, gains expected from using genomics could be lost due to bias caused
44 by pre-selection. The animal model, in this situation, has become obsolete,
45 and should be replaced.

46 Schaeffer (2011) and Goddard (2011) proposed replacing the animal model
47 with a SNP genotype effects model (SGEM). SGEM have been published by
48 various authors since 2001, but now there is a pressing need to use them in
49 place of animal models. In a SGEM attention is on the unbiased estimation
50 of SNP genotype effects rather than on animal breeding values. The SGEM is
51 the same as an animal model, except the many hundreds of thousands animal
52 additive genetic values are dropped and replaced by 50,000 (or fewer) SNP
53 genotype additive effects, where the SNP genotypes (coded as 1, 0, or -1) are
54 used as covariates. A problem is that not all animals with records (data) have
55 been genotyped, and thus, SNP genotypes have to be predicted for all animals
56 in the pedigree, using a model like that of Gengler (2007,2008) or Mulder et
57 al.(2010).

58 The purpose of this paper is to illustrate all of the calculations through
59 a small example that would be needed to apply a SGEM. Unbiased GEBV
60 can be obtained for each animal after unbiased SNP genotype effects have
61 been estimated. They can also be used to estimate genetic variances and
62 covariances among traits.

63 **Material and Methods**

64 **Data**

65 Example data are given in Table 1. There are 28 animals, of which 20
66 have observations and 14 have been genotyped for 7 SNPs. Normally, there
67 would be hundreds of thousands of animals with observations, and perhaps
68 30,000 would be genotyped for 50,000 SNPs. In this example, the 7 SNPs are
69 assumed to account for all of the genetic effects in the trait (which is how the
70 example was constructed). Four animals are inbred. Two traits have been
71 observed, but each trait will be processed separately.

72 *Table 1 goes here.*

73 **Prediction of Marker Genotypes**

74 The first step in the procedure is predict SNP genotypes for animals that
75 have not, or could not, be genotyped.

76 Gengler et al. (2007, 2008) and Mulder et al. (2010) have used an animal
77 model applied to the genotypes (-1, 0, or 1) of genotyped animals with an
78 overall mean, and an animal additive effect. The additive genetic relationship
79 matrix, \mathbf{A} amongst all animals with phenotypes and ancestors, is used, and a
80 very high heritability is assumed. The model for one marker at a time is

$$s_{ji} = \mu + g_i + e_i, \tag{1}$$

81 where

82 s_{ji} is marker j genotype, either -1, 0, or 1, for animal i ,

83 μ is an overall mean,

84 g_i is an animal's breeding value for the marker genotype, and

85 e_i is a residual error.

86 In terms of the example, \mathbf{g} is a vector of 28 by 1 for all of the animals
87 in Table 1. The observation vector is \mathbf{s} of 14 by 1 because only 14 animals
88 were genotyped, namely, 3, 7, 10, 11, 12, 15, 16, 17, 21, 22, 23, 25, 26, and 27,
89 and represents a column in Table 1 corresponding to one of the SNP markers.
90 Each marker is analyzed separately.

91 In matrix notation, for marker 1

$$\mathbf{s}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$
$$= \mathbf{1}\mu + \mathbf{Z}\mathbf{g} + \mathbf{e}$$

92 where

$$\mathbf{Z} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

93 The \mathbf{A} matrix is needed for all 28 animals, and this can be calculated
 94 using the tabular method. Knowing that

$$\mathbf{A} = \mathbf{TBT}'$$

95 where \mathbf{T} is lower triangular and \mathbf{B} is diagonal, the diagonals of \mathbf{B} are 1 for
 96 animals 1 through 6, and 0.5 for all other animals, except animals 22 and 23

97 which were 15/32 for each. These allow for the easy creation of \mathbf{A}^{-1} following
 98 Henderson's rules (1976).

$$\begin{aligned}
 E(\mathbf{g}) &= \mathbf{0} \\
 E(\mathbf{e}) &= \mathbf{0} \\
 Var(\mathbf{g}) &= Var \begin{pmatrix} \mathbf{g}_w \\ \mathbf{g}_o \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{ww} & \mathbf{A}_{wo} \\ \mathbf{A}_{ow} & \mathbf{A}_{oo} \end{pmatrix} \sigma_g^2 \\
 Var(\mathbf{e}) &= \mathbf{I} \sigma_e^2
 \end{aligned}$$

99 where \mathbf{g} is partitioned into animals with genotypes, \mathbf{g}_w , and animals without
 100 genotypes, \mathbf{g}_o . Let

$$\sigma_e^2 / \sigma_g^2 = 0.05 = \lambda,$$

101 which corresponds to a heritability of 0.9523. A heritability of 1 might cause
 102 computational problems.

103 The mixed model equations are

$$\begin{pmatrix} N & \mathbf{1}' & \mathbf{0}' \\ \mathbf{1} & \mathbf{I} + \mathbf{A}^{ww} \lambda & \mathbf{A}^{wo} \lambda \\ \mathbf{0} & \mathbf{A}^{ow} \lambda & \mathbf{A}^{oo} \lambda \end{pmatrix} \begin{pmatrix} \hat{\mu} \\ \widehat{\mathbf{g}}_w \\ \widehat{\mathbf{g}}_o \end{pmatrix} = \begin{pmatrix} \mathbf{1}' \mathbf{s} \\ \mathbf{s} \\ \mathbf{0} \end{pmatrix}. \quad (2)$$

104

105 The solutions for animals plus the overall mean gives a prediction of each
 106 animal's genotype. The predicted genotypes can be used directly in \mathbf{X} , as con-
 107 tinuous covariates, in Equation 3. Mulder et al.(2010) found a .69 correlation
 108 between predicted genotypes and actual genotypes in a simulation study. The
 109 result depends on how many animals were genotyped versus the number to
 110 be predicted. Each SNP marker would be analyzed separately. The predicted
 111 SNP genotypes can be used to analyze any trait, and only need to be calculated
 112 once. The results for animals in the example, $(\hat{\mu} + \widehat{\mathbf{g}})$, are in Table 2.

113 *Table 2 goes here.*

114 The predicted genotypes should be re-calculated whenever new animals
 115 are added to the pedigree, or whenever new animals have been genotyped.
 116 The predicted genotypes can be used in analyses of any trait.

117 In some cases animals might be genotyped with different SNP chips. The
118 SNP genotype effects should be on a set of SNPs that are common to each chip,
119 or which can be imputed from the various different SNP chips. Imputation
120 should be accomplished prior to predicting SNP genotypes for all animals.
121 This would allow all genotyped animals to participate in subsequent analyses.

122 One might choose to use a subset of SNPs, for example, only SNPs with
123 minor allele frequencies between 0.3 to 0.5, or SNPs that are distributed evenly
124 within and across chromosomes. Probably 5,000 to 50,000 SNP markers would
125 be sufficient. Studies are needed to determine an optimum number of markers.
126 However, the number of animals with records, N_r , and the number of geno-
127 typed animals, N_g , should be greater than the number of SNP markers, m .
128 ($N_r > N_g > m$)

129 **SNP Genotype Effects Model**

130 If animals have both phenotypic records and genotypes for markers, then
131 an appropriate linear model would be

$$\mathbf{y} = \mathbf{W}\mathbf{c} + \mathbf{X}\mathbf{m} + \mathbf{e} \quad (3)$$

132 where

133 \mathbf{y} is a vector of observations,

134 \mathbf{c} is a vector of 4 contemporary group effects,

135 \mathbf{m} is a fixed vector of 7 marker additive effects,

136 \mathbf{e} is a random vector of residuals,

137 \mathbf{W} is the design matrix relating contemporary group effects to the observa-
138 tions, and

139 \mathbf{X} is a matrix containing marker genotypes (i.e. the results in Table 2),
140 corresponding to each observation.

141 The expectations of the random vectors and the variances are given below.

$$\begin{aligned} E(\mathbf{e}) &= \mathbf{0}, \\ \text{Var}(\mathbf{e}) &= \mathbf{I}\sigma_e^2. \end{aligned}$$

142 The equations to solve are

$$\begin{pmatrix} \mathbf{W}'\mathbf{W} & \mathbf{W}'\mathbf{X} \\ \mathbf{X}'\mathbf{W} & \mathbf{X}'\mathbf{X} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{c}} \\ \hat{\mathbf{m}} \end{pmatrix} = \begin{pmatrix} \mathbf{W}'\mathbf{y} \\ \mathbf{X}'\mathbf{y} \end{pmatrix}.$$

143 Contemporary groups could have been a random factor, and the model,
 144 in real life, needs a factor to account for time trends. The number of markers,
 145 m , should be greatly less than the number of animals, and therefore, should
 146 be solvable more quickly than an animal model.

147 This model is not biased by using animals that have been pre-selected
 148 because no animal additive genetic relationships have been utilized. Also, \mathbf{m}
 149 is a vector of fixed effects in the model. The emphasis is on the unbiased
 150 estimation of fixed marker genotype effects through regressions on predicted
 151 marker genotypes.

152 Each trait is analyzed separately. This simplifies the software that is
 153 needed to do the analysis, and reduces the amount of computer time for the
 154 analyses. That is, the contemporary group effects can be absorbed into the
 155 matrix of marker genotype effects, then the resulting matrix can be directly
 156 inverted and solved. No iteration procedure is necessary. The results for the
 157 two traits are given in Table 3.

158 *Table 3 goes here.*

159 The correlation between the SNP estimates for trait 1 with trait 2 was
 160 -0.29, and the correlation between phenotypes was -0.28.

161 Genomic Estimated Breeding Values

162 For the example, Equation 1 gives a system of order equal to 4 contem-
 163 porary group effects and 7 marker covariates. Thus, let $\hat{\mathbf{m}}$ consist of $\hat{m}_{i,j}$, the
 164 solution for the i^{th} trait and j^{th} marker. Let $\hat{\mathbf{g}}$ (from Table 2) consist of $\hat{g}_{j,k}$,
 165 the predicted genotype for the j^{th} marker genotype of the k^{th} animal. Then a

166 genomic EBV (GEBV) for the k^{th} animal and the i^{th} trait is calculated as

$$GEBV_{i,k} = \sum_{j=1}^7 \widehat{m}_{i,j} \times \widehat{g}_{j,k}.$$

167 Results are in Table 4.

168 *Table 4 goes here.*

169 The GEBV should become more accurate as more animals are genotyped
170 and phenotyped over time. The predictions of $\widehat{\mathbf{g}}$ will also become better as
171 more animals are genotyped.

172 The correlations of the GEBV for trait 1 with the phenotypes for trait 1
173 (for 20 animals with records) was 0.89, and for trait 2 was 0.70. The correla-
174 tions are less than unity because of the removal of contemporary group effects
175 from the phenotypes.

176 Of interest is the correlation between the GEBV and the EBV from an an-
177 imal model. The regular animal model EBV are given in the last two columns
178 of Table 4, from single trait analyses using heritabilities of 0.25 for trait 1 and
179 0.30 for trait 2. The correlation for trait 1 between GEBV and EBV was 0.80,
180 and for trait 2 was 0.56. There was no bias in the regular EBV from the animal
181 model because none was built into the example data.

182 Note also that the GEBV have a greater range of values than the EBV
183 for each trait.

184 **Discussion**

185 The animal model is suffering from biases due to pre-selection of young
186 bulls in dairy cattle. Attempts to adjust the animal model equations for pre-
187 selection result in adhoc questionable methods. Since 2011, proposals have
188 been made to abandon animal models and switch to a model that estimates
189 SNP genotype effects, but few efforts have been made in that direction.

190 The SGEM is simple from a statistical point of view, and easier than an
191 animal model from a computational point of view. While the SGEM given in
192 this paper is an additive genetic model, the model can be easily transformed
193 to include dominance effects at each marker. Instead of one value per SNP,
194 there would be three, one for each possible genotype. The next step would
195 be to include interactions among SNP markers for additive by additive effects.

196 The SGEM may become as complex as researchers dare to venture. With the
197 animal model, one is limited to additive genetic effects.

198 One of the limitations of SGEM is the fact that SNP genotypes need to be
199 predicted for every animal that have a record. However, this author contends
200 that some years into the future, every dairy cattle animal that is born will
201 be genotyped, and that the SNP genotypes will either be directly available or
202 imputed.

203 The estimated SNP genotype effects should be fairly constant from one
204 year to the next, as long as there are more animals with records than there are
205 SNP markers to estimate. There is no need to have reference sets of animals
206 and validation sets of animals with SGEM models.

207 If each country uses a SGEM and the same SNP markers, then $G \times E$
208 interactions can be studied using the estimated SNP genotype effects directly.
209 Genetic correlations between countries could also be estimated using those
210 solutions. GEBV may be calculated for each animal, sires and cows, within
211 each country using the estimated SNP genotype effects from each country,
212 without the need for SNP MACE (multiple across country evaluation) or any
213 MACE. Interbull would coordinate estimated SNP genotype effects and make
214 comparisons between countries.

215 The actual SGEM may differ from country to country accounting for the
216 little differences in data collection and factors within the country. For exam-
217 ple, adjustments for age and month of calving, days pregnant, year-months of
218 calving, and contemporary groups.

219 The SGEM allow the simple calculation of GEBV for all animals, and not
220 from a biased animal model. Rankings of animals should be more accurate,
221 and hence selection decisions would be better made.

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Table 1
Example Data To Illustrate Methods.
CG = Contemporary Groups

CG	Anim	Sire	Dam	SNP Markers							Trait 1	Trait 2
				1	2	3	4	5	6	7		
	1											
	2											
	3			1	0	-1	0	-1	0	1		
	4											
	5											
	6											
	7	1	5	0	0	1	-1	0	1	-1		
	8	2	6									
1	9	1	8								14.8	11.0
1	10	1	8	0	-1	0	0	0	0	1	7.3	7.2
1	11	1	5	0	0	1	-1	1	1	0	6.1	18.4
1	12	2	6	-1	0	0	1	-1	0	0	17.5	8.2
1	13	2	7								22.5	6.2
1	14	2	7								10.9	9.0
2	15	2	9	-1	0	1	0	-1	0	1	4.2	6.7
2	16	2	10	1	-1	-1	1	0	-1	0	17.4	8.5
2	17	2	11	0	0	0	0	1	0	-1	15.3	15.3
2	18	3	5								17.5	12.5
2	19	3	6								9.0	17.9
3	20	1	9								10.0	1.7
3	21	2	10	0	-1	0	0	1	-1	0	10.1	9.1
3	22	3	15	0	0	0	0	-1	1	1	11.8	6.6
3	23	3	16	1	-1	-1	1	0	-1	-1	26.4	3.7
3	24	4	12								13.1	9.4
3	25	4	13	0	1	1	-1	0	0	-1	17.7	13.1
4	26	2	11	0	0	0	0	1	0	0	6.7	13.3
4	27	3	17	0	-1	-1	0	0	1	0	28.7	7.5
4	28	4	9								7.4	8.7

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Table 2
 Predicted Genotypes For All Animals

Anim	Predicted SNP Marker Genotypes						
	1	2	3	4	5	6	7
1	-0.0313	-0.2856	0.6008	-0.4591	0.4359	0.5017	0.2647
2	-0.1621	-0.2103	-0.3917	0.7955	0.3528	-0.9342	-0.3509
3	1.0390	0.0842	-0.9693	0.3478	-0.5457	0.0304	0.9795
4	-0.1798	0.3845	0.2185	-0.6223	-0.1306	-0.2626	-0.5750
5	-0.2561	-0.0845	0.2867	-0.7825	0.3021	0.3039	-0.9091
6	-0.4403	0.0810	0.2243	0.6900	-0.4450	0.3301	0.5602
7	0.0161	0.0655	0.8963	-0.7345	0.3365	0.8471	-0.9323
8	-0.3858	-0.3622	-0.1236	0.7074	-0.1762	-0.4001	0.4946
9	-0.5628	-0.2604	0.5479	-0.0126	-0.2724	0.1276	0.6879
10	0.1268	-0.8323	-0.0004	0.3407	0.4223	-0.0720	1.0013
11	-0.2287	-0.1890	0.6089	-0.9585	1.0345	0.5933	-0.2903
12	-1.2407	-0.2698	-0.4032	0.8843	-0.9448	-0.4575	-0.3087
13	0.0808	0.3636	0.6053	-0.0369	0.5231	0.0689	-0.6853
14	-0.0431	-0.0425	0.2822	0.0604	0.3745	-0.0136	-0.6117
15	-0.9380	0.0247	0.8297	0.2508	-0.6314	-0.1167	0.9181
16	0.8640	-0.9840	-1.0675	1.1482	0.2812	-1.1517	-0.0940
17	0.0764	0.0962	0.0101	0.3176	1.3577	0.0107	-0.8386
18	0.2581	-0.1335	-0.4746	-0.3507	-0.2552	0.0338	-0.0982
19	-0.1200	-0.3368	-0.7919	0.0995	-0.9148	-0.2391	0.3505
20	-0.1364	-0.1124	0.7350	-0.0752	0.2423	0.4753	0.6369
21	0.0113	-0.9033	-0.0963	0.2764	1.2299	-1.0678	0.0284
22	0.0000	0.0405	-0.1024	0.2346	-0.6942	0.7695	0.9762
23	0.9483	-0.9540	-1.1396	1.1393	0.2112	-1.1298	-0.9220
24	-0.3432	0.4244	0.2747	0.4980	-0.1707	0.0069	-0.0749
25	-0.2333	0.7546	0.6264	-0.9559	0.0617	-0.3634	-1.2092
26	-0.3051	-0.2652	-0.3688	-0.0828	0.9575	-0.4287	-0.3305
27	0.1800	-0.7313	-0.9147	0.3714	0.4389	0.9144	0.1217
28	-0.2171	0.2162	0.5374	-0.1633	-0.0473	0.0867	0.2106

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Table 3

Solutions from SGEM for example data.

Item	Trait 1	Trait 2
CG 1	14.337	13.358
CG 2	13.070	12.776
CG 3	14.264	9.221
CG 4	13.395	8.226
snp 1	-2.469	-2.000
snp 2	-6.542	10.721
snp 3	-7.353	-8.887
snp 4	-5.843	-3.921
snp 5	-7.073	5.600
snp 6	2.049	-2.037
snp 7	-10.701	4.603

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Table 4
GEBV from SGEM and EBV from animal model for animals in Example

Anim	Sire	Dam	Data.			
			GEBV		EBV	
			Trait 1	Trait 2	Trait 1	Trait 2
1	0	0	-2.7319	-4.1105	-1.9585	-0.6330
2	0	0	-0.3825	0.6667	0.6001	-1.5113
3	0	0	1.1782	6.8458	2.6518	-0.2479
4	0	0	-0.3985	2.8792	-0.6874	0.9594
5	0	0	2.4322	-1.9717	0.1473	1.2853
6	0	0	-0.0976	-4.3094	-0.7533	0.1475
7	1	5	9.0977	-8.8208	-0.4379	0.0985
8	2	6	-2.8221	-2.8908	-0.6517	-1.2368
9	1	8	-5.7706	-5.1327	-2.0477	-1.4907
10	1	8	-4.9474	-4.0074	-1.7126	-1.4890
11	1	5	-6.9651	0.7668	-1.7672	1.9735
12	2	6	2.2444	-5.0614	0.3538	-0.8983
13	2	7	4.1583	-2.6375	1.4513	-0.9989
14	2	7	4.3576	-4.0771	-0.3537	-0.8692
15	2	9	-9.0638	-5.4972	-1.9913	-2.3189
16	2	10	2.3673	-3.8338	0.8522	-2.2174
17	2	11	2.4020	2.6446	0.7022	0.4658
18	3	5	1.8052	2.4371	1.9409	0.3842
19	3	6	0.5403	1.2682	0.3406	0.8686
20	1	9	-4.2512	-4.6216	-2.4174	-2.0047
21	2	10	-3.6847	-1.0145	-1.1629	-1.0598
22	3	15	-2.8965	-0.7468	-0.1337	-1.3204
23	3	16	11.9437	-7.2520	3.0689	-1.7633
24	4	12	0.9230	-1.0911	-0.4006	0.2537
25	4	13	1.4814	3.0000	0.7269	0.8652
26	2	11	-7.8608	7.0972	-1.5942	0.8065
27	3	17	13.5606	-1.1473	3.4863	-0.3176
28	4	9	-3.0845	-1.1268	-2.1662	-0.4144

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